



Christ Church
Grammar School

2020
TEST 4

MATHEMATICS METHODS Year 11

Section One:
Calculator-free

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Working time for this section: 25 minutes
Marks available: 20 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

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2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens.
3. Answer all questions.
4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
6. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1

(5 marks)

- (a) Express the following in scientific notation.

(2 marks)

$$(6.3 \times 10^5) \div (3.15 \times 10^{-2})$$

$$= \frac{6.3 \times 10^5}{3.15 \times 10^{-2}}$$

$$= 2 \times 10^7$$

determines
the correct
constant

determines the
correct power of 10.

- (b) Simplify the following and express with positive indices where required.

(3 marks)

$$\frac{(16y^3z)^{\frac{1}{2}}}{8\sqrt{yz^{-3}}}$$

$$= \frac{16^{\frac{1}{2}} y^{\frac{3}{2}} z^{\frac{1}{2}}}{8 y^{\frac{1}{2}} z^{-\frac{3}{2}}}$$

distributes the power of $\frac{1}{2}$ correctly
to the indices in the numerator

$$= \frac{4 y^{2\frac{1}{2}} z^{\frac{4}{2}}}{8}$$

$$= \frac{yz^2}{2}$$

obtains the correct positive
indices for y, z.
obtains the correct factor of 2
in the denominator.

Question 2

(5 marks)

Solve the following equations.

(a) $\frac{1}{x^2} = 0.25$

(2 marks)

$\frac{1}{x^2} = \frac{1}{4}$

$4 = x^2$ ✓

$\therefore x = \pm 2$ ✓

re-writes the equation to obtain $x^2 = 4$

obtains both solutions $x = \pm 2$.

Note: Answer only is 2 marks
 $x = 2$ 0 marks without $x^2 = 4$.

(b) $(3^x)^2 + 6(3^x) - 27 = 0$

(3 marks)

let $y = 3^x$

$y^2 + 6y - 27 = 0$

$(y + 9)(y - 3) = 0$ ✓

$y = -9$ or 3

factorises expressions and has = 0.

or $(3^x + 9)(3^x - 3) = 0$ ✓

$3^x + 9 = 0$

or $3^x - 3 = 0$

$3^x = -9$

$3^x = 3$

no solution ✓

$\therefore x = 1$

If $y = -9$

then

$3^x = -9$

no solution ✓

states $3^x = -9$

has

no

solution

If $y = 3$

then

$3^x = 3$

$\therefore x = 1$ ✓

obtains

solution

$x = 1$

states

$3^x = -9$

has no

solution

obtains

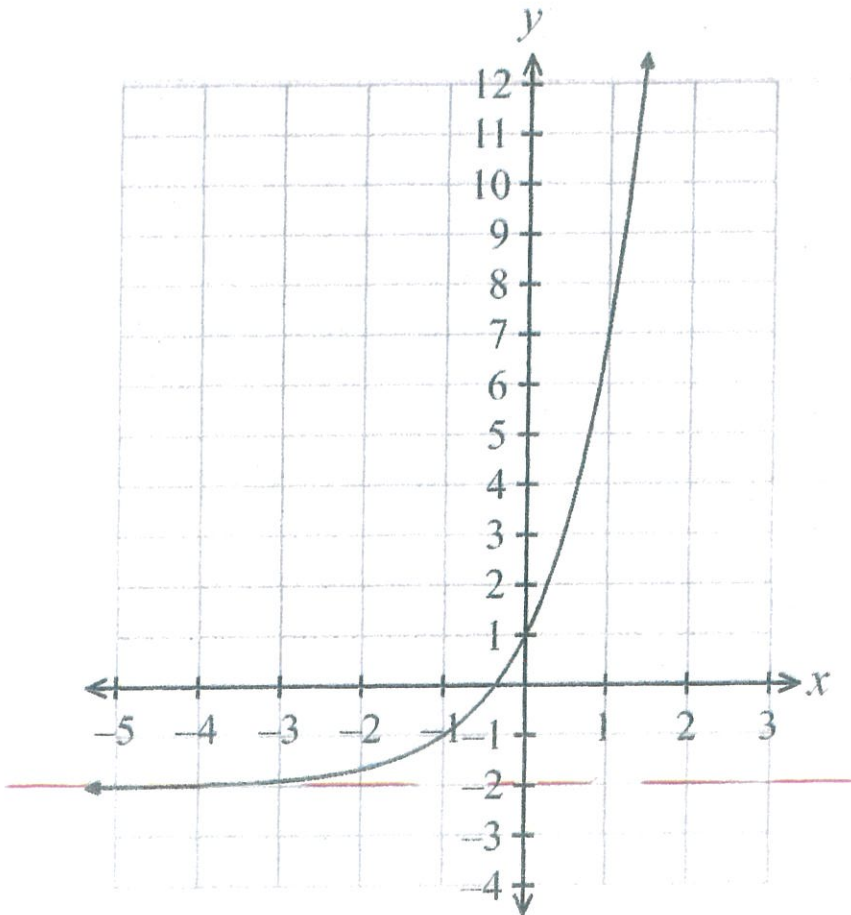
solution

$x = 1$ ✓

Question 3

(3 marks)

The graph of $y = a^{x-c} + b$ is shown below.



Determine the values of the constants a , b , and c .

$y \rightarrow -2$ as $x \rightarrow -\infty \therefore b = -2$

$y = a^{x-c} - 2$

when $x = -1, y = -1$

$-1 = a^{-1-c} - 2$

when $x = 0, y = 1$

$1 = a^{-c} - 2$

$1 = a^{-1-c}$

$3 = a^{-c}$

\therefore power must be 0

$3 = a^{-(-1)}$

$-1 - c = 0$

$3 = a^1$

$c = -1$

$\therefore a = 3, b = -2, c = -1$

Note: Can get FT marks for a and c if working clear.

See next page

obtains values for $a = 3, b = -2, c = -1$. Must state values explicitly.

Question 4

(4 marks)

The rising water level in a storm drain after a thunderstorm is measured and given by the equation $H = \frac{1}{8}(2^{t+1}) + 3$, where H is the depth of the water in the storm drain in millimetres, t minutes after the thunderstorm began.

- (a) Determine the depth of the water 2 minutes after the thunderstorm began.

(1 mark)

$$\begin{aligned} t=2 \quad H &= \frac{1}{8}(2^3) + 3 \\ &= \frac{1}{8} \times 8 + 3 \\ &= 4 \text{ mm} \end{aligned}$$

✓ states the correct answer.

- (b) Determine the amount of time that the height of the water in the storm drain is less than 11 mm.

(3 marks)

$$H < 11.$$

$$\text{Solve for } H = 11$$

$$11 = \left(\frac{1}{8}\right)2^{t+1} + 3$$

$$8 = \left(\frac{1}{8}\right)2^{t+1}$$

$$64 = 2^{t+1} \quad \checkmark$$

$$64 = 2^t \cdot 2$$

$$32 = 2^t$$

$$2^5 = 2^t$$

$$\therefore t = 5 \quad \checkmark$$

substitutes $H = 11$
and obtains
 $64 = 2^{t+1}$

solves equation correctly
for t .

\therefore The height of the water in the storm drain is less than 11 mm when $0 < t < 5$ mins (or when $t < 5$ mins).

✓ states final solution with correct statement about time.

Additional working space

Question number: _____



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TEST 4

MATHEMATICS METHODS Year 11

Section Two:

Calculator-assumed

Your name SOLUTIONS

Teacher's name _____

Time and marks available for this section

Working time for this section: 25 minutes

Marks available: 21 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in this assessment

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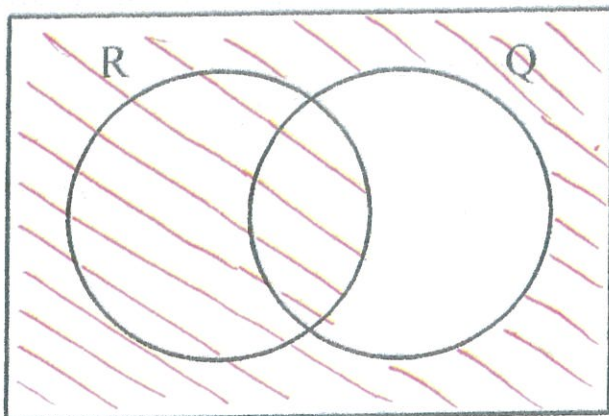
Question 6

(3 marks)

Shade the region indicated by

(a) $R \cup \bar{Q}$

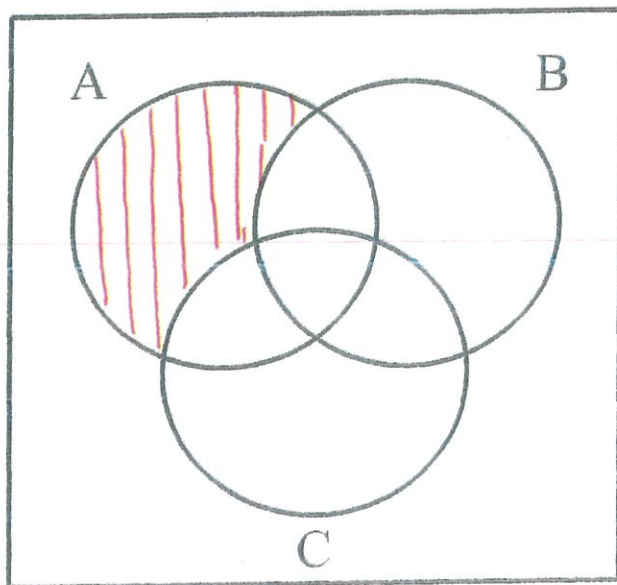
(1 mark)



✓ shades correct region

(b) $A \cap (\overline{B \cup C})$

(2 marks)



✓ identifies region $\overline{B \cup C}$
 ✓ shades correct final region $A \cap \overline{B \cup C}$

Question 7

(8 marks)

The following information regarding events A and B is known.

$$P(A) = \frac{3}{10}$$

$$P(B) = \frac{1}{2}$$

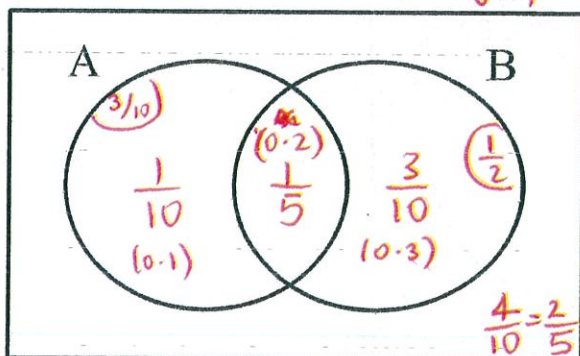
$$P(A|B) = \frac{2}{5}$$

(a) Complete the Venn diagram below.

(2 marks)

calculates $A \cap B$ correctly ✓

completes all other regions. ✓



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{1}{2} \times \frac{2}{5} = P(A \cap B)$$

$$P(A \cap B) = \frac{1}{5}$$

(b) Determine

(i) $P(\bar{A})$

(1 mark)

$$= \frac{7}{10} \quad (0.7) \quad \checkmark$$

obtains correct value

(ii) $P(A \cap B)$

(1 mark)

$$= \frac{1}{5} \quad (0.2) \quad \checkmark$$

obtains correct value

(iii) $P(A \cap \bar{B})$

(1 mark)

$$= \frac{1}{10} \quad (0.1) \quad \checkmark$$

obtains correct value

Question 7 continued

(c) Justify if the events A and B are mutually exclusive.

(1 mark)

A and B are not mutually exclusive
as $P(A \cap B) \neq 0$. ✓

correct statement
and must have
 $P(A \cap B) \neq 0$.

(d) Justify if the events A and B are independent.

(2 marks)

If independent then

$$P(A) \times P(B) = P(A \cap B)$$

$$\therefore \text{LHS} : P(A) \times P(B)$$

$$= \frac{3}{10} \times \frac{1}{2}$$

$$= \frac{3}{20}$$

✓ evaluates
 $P(A) \times P(B)$
correctly

$$\text{RHS} : P(A \cap B)$$

$$= \frac{1}{5}$$

$$\therefore P(A) \times P(B) \neq P(A \cap B)$$

$$\text{as } \frac{3}{20} \neq \frac{1}{5}$$

shows $P(A) \times P(B)$
 $\neq P(A \cap B)$

✓ and makes
final statement.

\therefore Events A and B are not independent.

Note: Cannot be awarded 2 marks if just state not independent,
must justify

See next page

Can use other independent rules to show.

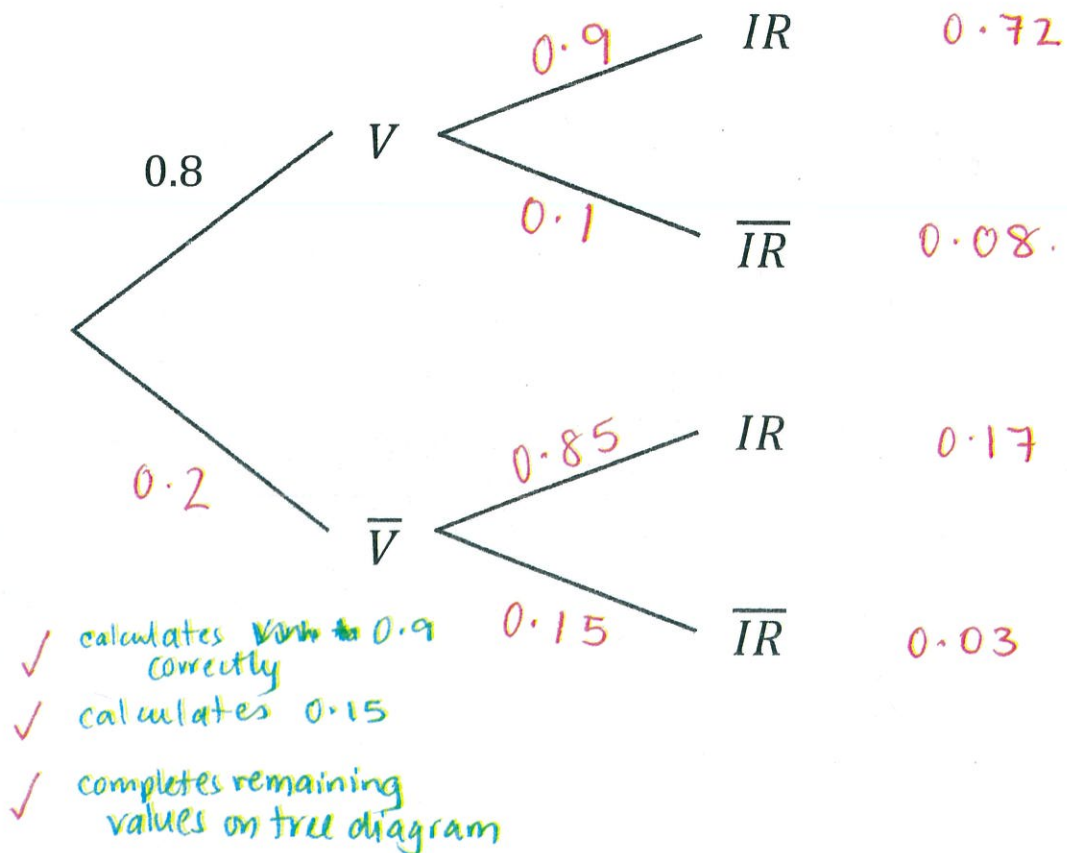
Question 8

(10 marks)

A DIY robot assembly kit has two main electronic features that can either work or be defective because of operator error when assembled. The features are Voice Recognition Command (V) and Infra-red Obstacle Avoidance (IR). The Voice Recognition Command feature works 80% of the time. The probability that both will work is 0.72 and the probability that both will be defective is 0.03.

(a) Complete the tree diagram with the correct probabilities on each branch.

(3 marks)



(b) Determine the probability that the Infra-red Obstacle Avoidance works. (2 marks)

$$\begin{aligned}
 & P(V \cap IR) + P(\overline{V} \cap IR) \quad \checkmark \quad \text{identifies correct probabilities to add} \\
 = & 0.72 + 0.17 \\
 = & 0.89 \quad \checkmark \quad \text{obtains solution Probability is 0.89}
 \end{aligned}$$

Question 8 continued

(c) Determine the probability that only one feature works.

identifies 1-given probability

(2 marks)

identifies correct probabilities and add them

$$P(V \cap \bar{IR}) + P(\bar{V} \cap IR) \\ = 0.08 + 0.17 \\ = 0.25 \quad \checkmark$$

$$\text{or } 1 - P[(\bar{V} \cap \bar{IR}) + (V \cap IR)] \\ = 1 - (0.72 + 0.03) \\ = 0.25 \quad \checkmark$$

obtains correct answer

obtains correct answer

(d) Given that only one feature works, justify whether it is more likely that the Voice Recognition Command or the Infra-red Obstacle Avoidance is the component that works.

(3 marks)

$$P(\bar{V} | IR) = \frac{P(\bar{V} \cap IR)}{P(IR)} \\ = \frac{0.17}{0.17 + 0.72} \\ = \frac{17}{89} \\ = 0.191 \quad \checkmark$$

Note: If they calculate $P(V | \bar{IR})$ and $P(IR | \bar{V})$ only $\frac{1}{3}$ if correct comparison.

evaluates probability that one works $P(\bar{V} | IR)$

$$P(\bar{IR} | V) = \frac{P(\bar{IR} \cap V)}{P(V)} \\ = \frac{0.08}{0.8} \\ = 0.1 \quad \checkmark$$

evaluates probability that the other works $P(\bar{IR} | V)$

\therefore It is more likely the infra-red obstacle avoidance component works as $P(\bar{V} | IR) > P(\bar{IR} | V)$
($0.191 > 0.1$)

Final statement with numerical/probability comparison.

End of questions

